Formula Sight Reduction Method

Scenario
D.R. Position:  51° 54’N 21° 55’W.
Date: 18 July 2009
Zone Time: 16h 44m (+1)
DWT:  17h 50m 28s
DWE 40s fast
Body observed:  Sun L.L.
Index error:  +0’.54
Ht. of eye:  8m.
Temperature:  28°C. Pressure:  991mb.

Sight Reduction Form for use with Cosine Formula Method

<table>
<thead>
<tr>
<th>Observation Notes.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date: 18 July, 2009</td>
</tr>
<tr>
<td>DR Pos: 51°54’N, 21°55’W</td>
</tr>
<tr>
<td>Zone: +1h</td>
</tr>
<tr>
<td>Zone Time: 16h 44m</td>
</tr>
<tr>
<td>Ht: 8m.</td>
</tr>
<tr>
<td>IE: +0’.54</td>
</tr>
<tr>
<td>DWT: 17h 50m 28s DWE: -40s</td>
</tr>
<tr>
<td>Temp: 28°C. Pressure: 991mb.</td>
</tr>
<tr>
<td>Body Observed: Sun L.L.</td>
</tr>
<tr>
<td>Sext. Alt: 32° 10.’4 Compass bearing: 261°</td>
</tr>
</tbody>
</table>

Step 1. Convert DR lat and long to decimals.
Lat: 51°.9N Long: 21°.9W
(Assumed positions are not used with formula method)

Step 2. Calculate PZ. (90 - Lat).
PZ = 38.1

Step 3. Calculate Greenwich Date at time of observation.
Date: 18 July, 2009
Zone time: 16h 44m
Zone correction: +1h
Universal Time (GMT): 17h 44m
Deck watch time: 17h 50m 28s
Deck watch error: -40s
Step 4. Calculate Greenwich Hour Angle and Declination.

Date:                    GHA               Dec
UT:  17h 73° 26’.1        N20° 54’.7 (d:0’.5)
Inc:  49° 48’ +12° 27’.0 -0’.4
                                85° 53’.1 N20° 54’.3
Decimalize: 85°.885         N20°.9

Step 5. Determine if Lat and Dec are 'Same' or 'Contrary'.
Lat:  51°.9N                Dec: N20°.9
Same / Contrary (select)

Step 6. Calculate PX. (90 - Dec if same) (90 + Dec if contrary).
PX = 69°.1

Step 7 Calculate LHA: Long East, LHA = GHA + LONG (- 360° as necessary)
                        Long West, LHA = GHA - LONG (+ 360° as necessary)
GHA: 85°.885
DR Long: 21°.9W
LHA= 63.98

Step 8. Determine Angle ZPX.
ZPX = LHA = 63.98

Step 9. Calculate True Altitude at True Position
Sextant Altitude: 32° 10’.4
Index error (IE): +0’.54
Observed Altitude: 32° 10’.94
Dip (ht. 8m.): -5’.0 (table 6a)
Apparent Altitude: 32° 05’.94
Altitude correction: +14’.50 (table 6d)
Added refraction: +0’.10 (table 6c)
True Altitude: 32° 20’.54
Decimalize: 32°.342
Note compass bearing at time of observation: 261°

Step 10. Calculate Zenith Distance at True Pos. (90° - Altitude).
Zenith Dist = 90° - 32°.342 = 57°.658

Step 11. Calculate Zenith Distance at DR Position. (ZX).
ZPX : 63.98 (From Step 7)
PZ : 38.1 (From Step 2)
PX : 69°.1 (From Step 6)
Reminder: The formula for calculating Zenith Distance (ZX) is:
\[
\cos(ZX) = [\cos(PZ) \times \cos(PX)] + [\sin(PZ) \times \sin(PX) \times \cos(ZPX)]
\]
Substituting the values of PZ, PX, and ZPX in this formula, we have:
\[
ZX = [\cos(38.1) \times \cos(69.1)] + [\sin(38.1) \times \sin(69.1) \times \cos(63.98)]
\]
\[
= [0.7869 \times 0.3567] + [0.6170 \times 0.9342 \times 0.4387]
\]
\[
= 0.2807 + 0.2528
\]
\[
= 0.5335
\]
\[
ZX = \cos^{-1}(0.5335) = 57.7577
\]
\[\therefore\text{Zenith Distance at DR position} = 57.7577\]

Step 12. Calculate Azimuth Angle at DR Position (PZX)

\[\begin{align*}
PZ &= 38.1 \quad \text{(From Step 2)} \\
PX &= 69.1 \quad \text{(From Step 6)} \\
ZX &= 57.7577 \quad \text{(From Step 11)}
\end{align*}\]

Reminder: The formula for calculating azimuth angle (PZX) is:
\[
\cos(PZX) = \cos(PX) - \left[\cos(ZX) \times \cos(PZ) \right] - \left[\sin(ZX) \times \sin(PZ)\right]
\]
Substituting the values of PZ, PX and ZX in the above formula, we have:
\[
PZX = \cos(69.1) - \left[\cos(57.7577) \times \cos(38.1) \right] - \left[\sin(57.7577) \times \sin(38.1)\right]
\]
\[
= \frac{0.3567 - [0.5335 \times 0.7869]}{0.8458 \times 0.6170}
\]
\[
= \frac{0.3567 - 0.4198}{0.5219}
\]
\[
= \frac{-0.0631}{0.5219}
\]
\[
= -0.1209
\]
\[
PZX = \cos^{-1}(-0.1209) = 96.944
\]
\[\therefore\text{Calculated azimuth Angle at DR position} = 096.944\]

Step 13. Convert azimuth angle (Z) to true azimuth (ZN):

<table>
<thead>
<tr>
<th>Rules for converting Azimuth Angle (Z) to True Azimuth (Zn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lat. North</td>
</tr>
<tr>
<td>LHA&gt;180°</td>
</tr>
<tr>
<td>LHA&lt;180°</td>
</tr>
<tr>
<td>DR Lat:</td>
</tr>
<tr>
<td>Azimuth Angle (Z):</td>
</tr>
<tr>
<td>LHA :</td>
</tr>
<tr>
<td>ZN = 360° - 96.944 = 263°</td>
</tr>
<tr>
<td>Therefore calculated true azimuth at DR position = 263°</td>
</tr>
</tbody>
</table>

Reminder: Subtract the ZD at the true position (a) from the ZD at the DR position (b).
- If the result is positive, the intercept is towards the azimuth.
- If the result is negative, the intercept is from the azimuth.

\[\begin{align*}
a. \text{Zen. Dist. at DR Pos} &= 57°.7577 \quad \text{(from step 11)} \\
b. \text{Zen. Dist. at True Pos} &= 57°.658 \quad \text{(from step 10)}
\end{align*}\]
<table>
<thead>
<tr>
<th>Intercept: $a - b = 0^\circ.0997$</th>
<th>Convert to minutes: $5.982'$ (multiply by 60)</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Azimuth: $263^\circ$</td>
<td></td>
</tr>
<tr>
<td>Intercept: $5.982'$ to $263^\circ$</td>
<td></td>
</tr>
</tbody>
</table>

**Step 15. Plot the position line.**
(Reminder: Plot intercept from DR position along azimuth line).

**DR Lat:** $51^\circ.9N$  **DR Long:** $21^\circ.9W$ (from step 1)
**Intercept:** $5.982'$ to $263^\circ$ (from step 14)